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Comparison of Convolutional Coupled Codes and Partially Systematic Turbo Codes for Medium Code Lengths

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Abstract — **Two classes of concatenated codes of rate 1/2 and medium code lengths (600, 2000) are considered: convolutional coupled codes and partially systematic turbo codes. As opposed to the classical turbo code (Berrou, Glavieux), which is a systematic code, coupled codes contain no systematic bits and partially systematic turbo codes only some systematic bits. These codes can still be iteratively decoded and they show even better distance properties. This leads to better error rate performances in a wide range of signal-to-noise ratios, especially in the "flattening region". The two code classes and their performances are presented and compared.**

I. Introduction

Since the first publications regarding turbo codes [1], the subject of soft-in soft-out iterative decoding of concatenated codes has received considerable attention. Turbo codes have near Shannon limit error correction performance and thus, they are very attractive for application to various areas in communications. In this paper two classes of concatenated codes are considered, the convolutional coupled codes and the partially systematic turbo codes.

The coupled codes were introduced in [2, 3]. The code is formed by ν identical systematic outer codes of rate k/n and minimum distance d_o and k systematic identical inner block codes with parameters $(2\nu, \nu, d_i)$. They are linked together ("coupled") such that only the systematic parts of the outer codes are encoded with the inner block encoders. The bits of each information word (info word) of the outer codes are scrambled by a given interleaving before entering the inner encoders. As opposed to the parallel concatenated (turbo) codes, in which the information bits (info bits) and the parity bits of the constituent codes are transmitted [1], only the redundancy produced by the outer codes and the inner block codes is transmitted. Therefore, the resulting code is non-systematic and the overall code rate remains k/n, which is the rate of the outer codes. This paper deals with convolutional coupled codes (CCC), in which convolutional codes are used as outer codes.

Partially systematic turbo codes (PSTC) were introduced in [4] for the case of large code lengths and uniform interleaving. The properties and the performance for medium code lengths and designed interleavers are investigated in this paper. PSTCs consist of two punctured parallel concatenated convolutional codes. As opposed to the classical systematic turbo code [1], not only the parity parts, but also the systematic parts are punctured. Thus, PSTCs can be seen as a generalization of the classical turbo codes. Since only a part of the information and parts of the redundancies are transmitted, the code is *partially systematic*.

Both the CCCs and the PSTCs are iteratively decoded by means of soft-in soft-out decoders for the constituent codes which exchange extrinsic information. The codes together with their iterative decoders yield higher coding gains than the classical systematic turbo code in a wide range of signal-to-noise ratios (SNR).

As the construction of CCCs is very similar to that of PSTCs, we present a comparison of these two code classes. Emphasis is given to code lengths in the range of 600 to 2000 code bits. Constituent codes of the same memory length and very similar interleaving schemes are applied to provide for a fair comparison. The PSTCs considered in this paper are derived from the turbo codes specified for the UMTS standard [5].

The paper is structured as follows: In Sections II and III the CCCs and the PSTCs are presented, respectively. Code structures, distance properties, and performances are described and discussed. The comparison of the two code classes with respect to the same aspects follows in section IV. The error rate plots for both code classes are arranged on one page at the end of the paper, so that they can be compared in a convenient way.

II. Convolutional Coupled Codes

A. Code Structure

The binary convolutional coupled code (CCC) is a coupled code where the systematic outer codes are rate $1/2$ recursive systematic convolutional (RSC) codes. Firstly, we will discuss the outer RSC code and the inner block code separately. Then, we will state the encoding scheme for the CCC.

The outer code words are terminated, i.e., we start encoding in the all-zero encoder state and ensure that after the encoding process, all memory elements contain zeros again. Due to the termination bits, the rate of the (terminated) code is no longer $1/2$, but is given as

$$R_o = \frac{1}{2}\frac{L}{L+m},$$

where L and m denote the info word length and the memory length, respectively.

The systematic inner block code is defined by the $\nu \times 2\nu$ encoding matrix $G = [I_\nu P]$, where I_ν is the $\nu \times \nu$ identity matrix and P a $\nu \times \nu$ matrix called *coupling matrix*. In this work we consider the encoding matrix

$$G = [I_\nu P] = \begin{bmatrix} \mathbf{1} & 0 & \cdots & 0 & \mathbf{0} & \mathbf{0} & 1 & \cdots & 1 \\ 0 & \mathbf{1} & \cdots & 0 & 1 & \mathbf{0} & \mathbf{0} & \cdots & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \mathbf{0} & \cdots & 1 \\ \vdots & & & & & & & & \\ 0 & 0 & \cdots & \mathbf{1} & \mathbf{0} & 1 & 1 & \cdots & \mathbf{0} \end{bmatrix} .$$

The entries of the coupling matrix P are 0 in the positions (i, j) if $j = i$ or $j = i + 1 \bmod \nu$, and 1 otherwise. Since P is a circulant matrix, the resulting block code is quasi cyclic. The number of ones in a row of the coupling matrix is called the *coupling factor* and is denoted by $\mathcal{K}$. Note that $\mathcal{K} = \nu - 2$. In order to make sure that only the all-zero code word has a parity part with weight 0, the number of rows ν has to be odd. It can be shown that the minimum distance of this block code is equal to 4 for all ν.

The encoding scheme for the CCCs is shown in Fig. 1. The overall info word $\boldsymbol{u}$ of length $K = \nu \cdot L$ is written in a $\nu \times L$ rectangular matrix. Each row of this matrix represents an info word for an outer RSC encoder. Extending each row with the m required termination bits leads to the $\nu \times k$ rectangular matrix I, $k = L + m$. The bits in the i-th row of matrix I are fed into the corresponding RSC encoder and the resulting parity bits are written into the i-th row of matrix $\mathcal{R}_o$, $i = 1, 2, \ldots, \nu$. After encoding with the ν outer codes, intra-row permutations are applied to the bits in matrix I according to the interleaving scheme described under II.B.

The number of inner block encoders is given by k. Let $\boldsymbol{u}^t = (u_1^t, u_2^t, \cdots, u_\nu^t)$ denote the t-th column of I and $\boldsymbol{v}_o^t = (v_{o,1}^t, v_{o,2}^t, \cdots, v_{o,\nu}^t)$ the t-th column of $\mathcal{R}_o$, $t = 1, 2, \ldots, k$. The word $\boldsymbol{u}^t$ is fed into the inner block encoder. The parity part of the resulting code word, $\boldsymbol{v}_i^t = (v_{i,1}^t, v_{i,2}^t, \cdots, v_{i,\nu}^t)$, is written into the t-th column of $\mathcal{R}_i$, $t = 1, 2, \ldots, k$.

The overall code word of the CCC is defined as $\boldsymbol{c} = (\boldsymbol{v}_i^1 \boldsymbol{v}_o^1, \boldsymbol{v}_i^2 \boldsymbol{v}_o^2, \cdots, \boldsymbol{v}_i^k \boldsymbol{v}_o^k)$. Since only the two parts of redundancy $\mathcal{R}_o$ and $\mathcal{R}_i$ (see Fig. 1) produced by the outer RSC codes and the inner block codes, respectively, are transmitted, the CCC is *non-systematic*.

B. Interleaving

If only two outer code words are non-zero, the worst case occurs if these two code words are identical and if additionally they are of minimum weight. I.e., the weight of each non-zero column in matrix I is two and hence produces an inner redundancy of only weight two. Using an appropriate interleaving scheme, these cases of two low-weight outer information vectors that lead to only weight two inner information vectors may be avoided. Consequently, the number of low-weight CCC words, which dominates the performance in terms of bit error probability, is reduced. This can be accomplished by employing *modulo interleavers* [3] defined as follows.

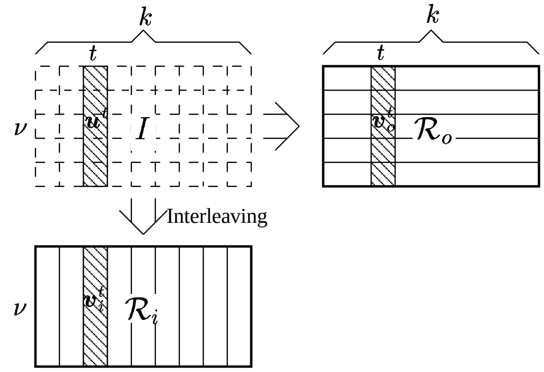

Fig. 1: Encoding scheme for convolutional coupled codes (CCC).

Let $(u_0, u_1, \ldots, u_{k-1})$ denote a row of matrix I and let g, $0 < g < k$, be a number which is relatively prime to k. Then, the modulo-interleaved row is defined by $(u_{\pi_g(0)}, u_{\pi_g(1)}, \ldots, u_{\pi_g(k-1)})$ with

$$\pi_g(i) = i \cdot g \bmod k, \quad 0 \leq i \leq k - 1.$$

Now, let g_i denote the value applied for permuting the i-th row, $i = 1, \ldots, \nu$. The resulting permutations are pair-wise different if the values g_i are pair-wise different and if they fulfill the condition

$$g_j \cdot g_i^{-1} \neq 1 \bmod k, \quad i, j = 1, \ldots, \nu,$$

which can easily be proofed by group theory.

By using these pair-wise different row-permutations, the number of cases, explained above, are greatly reduced. This results in good distance properties of the CCCs.

C. Distance Properties and Performances

The simulated bit error rate (BER) performance of a rate $1/2$ CCC with $\nu = 7$ outer RSC codes (CCC_7) of memory 3 is shown in Fig. 2. The generator polynomial of the outer RSC codes is $g(D) = (1 + D + D^3)/(1 + D + D^2 + D^3)$. The simulation is done for the binary-input additive white Gaussian noise (AWGN) channel. The bit error rate curve of the CCC is similar to that of turbo codes and can be divided into two regions: the "waterfall region", which appears at smaller SNR and has a steep slope, and the "flattening region", which appears at higher SNR and is caused by code words of small weight.

In the following, the BER and the minimum distance of the code CCC_7 will be lower-bounded by means of the subcode CCC_7^{bd}.

Let $\boldsymbol{c}_i = (\boldsymbol{u}_i, \boldsymbol{v}_i)$ be an outer RSC code word, where $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ correspond to the info word and the parity word, respectively. If all other $(\nu - 1)$ RSC code words are zero, the weight of the resulting CCC word $\boldsymbol{c}$ can be calculated as

$$w(\boldsymbol{c}) = \mathcal{K} w(\boldsymbol{u}_i) + w(\boldsymbol{v}_i) \,,$$

where $w(.)$ denotes the Hamming weight of a vector and $\mathcal{K}$ is the coupling factor.

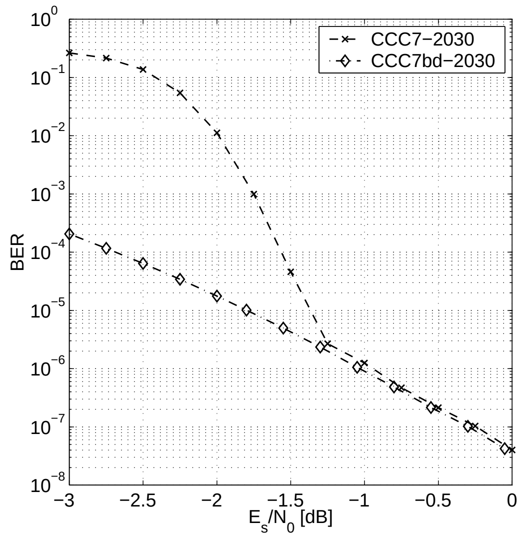

Fig. 2: Bit error rates of the convolutional coupled code CCC_7 (rate $R_c \approx 1/2$, code length $N = 2030$) and of its subcode CCC_7^{bd}.

For further analysis, we introduce the modified active row distance: Consider an RSC code of rate $1/2$. Let $\boldsymbol{c} = (c_1, c_2, \cdots)$ denote a code word, where $c_t = (u_t, v_t)$ is a code block of length 2, comprising one info bit u_t and one parity bit v_t. Let further denote σ_t the state at time instance t of the path which belongs to the code word $\boldsymbol{c}$. The *modified active row distance* at position j is defined as

$$d_j^{r,m} = \min_{\boldsymbol{c} \in \gamma_j} \sum_{t=0}^{j-1} (\mathcal{K} w(u_t) + w(v_t)) \,, \qquad (1)$$

$$\gamma_j = \{\boldsymbol{v} \in C |\ \sigma_0 = \sigma_j = 0, \sigma_t \neq 0 \quad \text{if} \ \ \sigma_{t-1} = 0 \ \forall\, t < j\} \,.$$

Note that info bits are weighted with the coupling factor $\mathcal{K}$ and parity bits are weighted with one.

The subcode CCC_7^{bd} is defined as the CCC CCC_7 with only one of the $\nu = 7$ outer codes being different from zero. Consequently, the minimum distance of the code CCC_7^{bd} can be given by the minimum of the modified active row distances according to (1), i.e.,

$$d_{min}^{bd} = \min_{j=1,2,\dots} \{d_j^{r,m}\} \qquad (2)$$

with $\mathcal{K} = 5$. The minimum distance of CCC_7^{bd} was found to be 16 by computer search.

Decoding the subcode CCC_7^{bd} can be interpreted as decoding the CCC CCC_7 with the additional a-priori knowledge that $\nu - 1$ outer codes are zero. Therefore we can consider the performance of CCC_7^{bd} as a lower bound for performance of CCC_7.

Fig. 2 shows that for higher SNR the performance curve of CCC_7 gets very close to that obtained for CCC_7^{bd}. This convergence gives a heuristic evidence for the fact that the minimum distance properties of CCC_7 and CCC_7^{bd} are similar. Thus, the minimum distance of the CCC can be estimated by means of (2). We call d_{min}^{bd} the *effective free distance* and CCC_ν^{bd} the *effective boundary coupled code*. Further results about the distance properties are propounded in [6].

From (1) and (2) follows that increasing the coupling factor $\mathcal{K}$ leads to a higher effective free distance. In order to investigate this, we consider the convolutional coupled codes CCC_5, CCC_7, and CCC_9, obtained from the constructions with 5, 7, and 9 outer RSC codes, respectively. The simulation results for the code lengths $N \approx 600, 2030$ are presented in Fig. 4. We observe that increasing ν leads to lower bit error rates at moderate and high SNR. Since codes with large minimum distance perform asymptotically better than codes with small one, we can conclude that increasing the number ν of outer codes improves the distance properties of the CCC.

Let us now consider the performance of CCCs of different code lengths. Fig. 4 shows also that enlarging the code length leads to a performance gain in a wide range of bit error probabilities. This confirms the fact that increasing the interleaver length for a given concatenated code leads to better performance. Nevertheless the "flattening region" is dominated by the number of outer codes ν.

Fig. 5 shows the word error rate (WER) performance of some selected CCCs. The behavior is quite similar to that of the BER curves. Increasing the code length N results mostly in an improvement for lower SNR, and increasing the number ν of outer codes leads to an improvement for higher SNR.

We conclude that the code length and the number of outer codes ν have to be traded off according to the system requirements.

III. Partially Systematic Turbo Codes

A. Code Structure

The encoder of the partially systematic turbo code (PSTC) is depicted in Fig. 3. The info word $\boldsymbol{u}$ of length K is encoded by two RSC encoders of memory length m. The first one (RSC1) is of rate $1/2$ and its code word comprises both the systematic word $\boldsymbol{u}$ and the parity word $\boldsymbol{p}_1$; the second one (RSC2) is of rate 1 and its code word comprises only the parity word $\boldsymbol{p}_2$. Before entering the second encoder, the info word is interleaved according to III.B. Both RSC encoders are terminated by means of post-interleaver flushing, i.e., they are driven back to the zero states, but the termination bits of each encoder are not fed to the respective other encoder [5]. The turbo code word $\boldsymbol{c} = (\boldsymbol{u}, \boldsymbol{p}_1, \boldsymbol{p}_2)$ consists of the systematic part and the two parity parts, and the code rate is given by $K/(3K + 3m) \approx 1/3$. This code is regarded as a mother (turbo) code in the following.

In order to raise the code rate to about $1/2$, this mother code is punctured. Classically [1], only the parity words, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, are punctured such that every second parity bit is transmitted (this corresponds to P_1 in Table I). In [4] it has been shown that spreading the puncturing over both the systematic and the parity parts improves the distance properties of the punctured code.

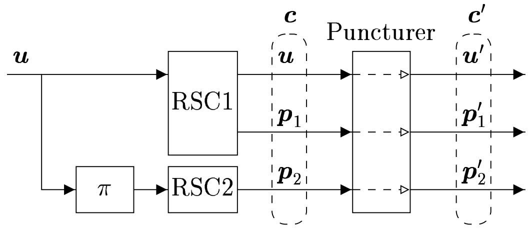

Fig. 3: Encoder for partially systematic turbo codes (PSTC).

Since the resulting code words do not contain all of the systematic bits any more, the codes are denoted as *partially systematic* turbo codes.

The puncturing is designed according to two criteria: (i) the code rate of the PSTC is $R_c \approx 1/2$; (ii) the two parity words are punctured equally strong. Let ρ_u denote the ratio of the number of systematic bits in the PSTC and those in the mother turbo code (see Fig. 3). Then, ρ_u describes how "systematic" a code is. Some examples for ρ_u and corresponding puncturing patterns are listed in Table I. Note that $\rho_u = 1$ corresponds to the classical (systematic) turbo code and $\rho_u = 0$ to a non-systematic turbo code.

ρ_u	1	$3/4$	$1/2$	0
P_{ρ_u}	$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

TABLE I

Puncturing matrices P_{ρ_u} for the construction of rate $1/2$ PSTCs. (0 means "punctured" and 1 "unpunctured".)

The code word of the PSTC is given as $\boldsymbol{c}' = (\boldsymbol{u}', \boldsymbol{p}_1', \boldsymbol{p}_2')$, where the prime denotes the respective punctured words (see Fig. 3).

For the comparison with the CCCs, we used as mother turbo code the one specified in the UMTS standard [5]. The two RSC encoders are of memory 3 and they use the generator polynomial $g(D) = (1 + D^2 + D^3)/(1+D+D^3)$. The interleaving scheme is described briefly in the next section. To generate the PSTCs with ρ_u, we applied the puncturing matrices P_{ρ_u} according to Table I. The code with P_1 corresponds to the classical (systematic) turbo code [1] and can be regarded as a reference.

B. Interleaving

A large variety of deterministic and stochastic methods for interleaver design are proposed in literature. To provide for easy reproducibility, a well defined deterministic interleaving scheme is adopted, namely the one specified for the UMTS turbo code. The general principles are shortly reviewed, for details we refer the reader to [5].

Firstly, the info bits are read into a rectangular matrix row-by-row. Then, three kinds of permutations are performed[1]: (i) the columns are permuted; (ii) each of the rows is modulo-permuted using a row-specific parameter; (iii) the rows are permuted. Finally, the matrix is read out column-by-column. Since permutation (ii) is almost identical to the one used for CCCs, the two interleaving schemes bear a strong similarity.

C. Performance

The error rate performance of rate $1/2$ PSTCs with $\rho_u = 1, 3/4, 1/2$ and code lengths $N = 606, 2006$ (info word lengths $K = 300, 1000$) for transmission over the binary-input AWGN channel was determined by simulation . (Although the PSTC with $\rho_u = 0$ has probably the best distance properties, iterative decoding does not succeed; therefore, this case is not considered.) For iterative decoding, a maximum number of 80 iterations was allowed and the stopping criterion proposed in [7] was applied. Between 6 and 20 iterations were used on average[2] depending on the code length and the value of ρ_u. In our experience, for smaller values of ρ_u more iterations are required. Note that the general structure of the iterative decoder (and thus the decoding complexity) does not depend on the puncturing pattern.

The simulation results for the BER and the WER are given in Fig. 6 and Fig. 7. Reducing the number of systematic bits (lowering ρ_u) results in lower error rates in the "flattening region", as could be expected from [4]. Especially for the shorter code, the WER shows a remarkable gain. The disadvantage is a "slight shift" of the "waterfall-region" to higher SNR. But this loss in power efficiency is only small if ρ_u is not too small. Therefore, puncturing the systematic bits can be regarded as a second means for lowering the "flattening region" besides interleaver design.

It can be observed that the performance degradation in the "waterfall-region" due to puncturing the systematic bits (lowering ρ_u) is lower for the WER than for the BER. That means that the average number of bit errors per erroneous word is larger for lower values of ρ_u. By employing a different encoding scheme, i.e. a different mapping of info words $\boldsymbol{u}$ onto PSTC words $\boldsymbol{c}'$, the BER may be reduced such that it shows the same behavior as the WER.

IV. Comparison

Both the convolutional coupled codes and the partially systematic turbo codes are constructed with memory 3 RSC codes and they use interleavers which have a quite similar structure. Both codes are iteratively decoded by exchanging extrinsic information between the decoders of the constituent codes. All of these codes are of rate $R_c \approx 1/2$ and code lengths $N \approx 600, 2000$.

Although the encoders and the decoders of these two code classes look similar, there are some remarkable differences:

[1]In [5], the permutations (i) and (ii) are performed in one step. However, the separation makes clearer which different kinds of interleaving this scheme comprises.

[2]The stopping criterion was *not* optimized with respect to the number of iterations.

- The RSC codes of the CCCs are coupled by the inner block codes, whereas those of the PSTCs are coupled by having encoded the same (interleaved) info bits.
- The structure of the CCCs allows to give a lower bound of the minimum distance, but no bounds are known for PSTCs at present[3].
- The PSTC may be improved with respect to the distance properties by decreasing the number of systematic bits (decreasing ρ_u), but a certain number of systematic bits are necessary for the iterative decoder to converge at low SNR (see Fig. 6, 7). The CCC does not contain any systematic bits; nevertheless, iterative decoding works.
- The delay caused by the interleaver length in CCCs amounts only $1/\nu$ of the delay generally caused in turbo codes, since the ν outer codes may be simultaneously decoded.
- For a fixed code length and a fixed generator polynomial of the RSC code, the performance of the PSTC is influenced by the interleaver structure and by the puncturing pattern, whose most distinguishing feature is the value of ρ_u. The performance of the CCC is subject to the number of outer and inner codes and to the (inner) block code.

In the following the codes are compared with respect to their error rate performances (see Fig. 4, 5, 6, and 7).

Firstly, enlarging the interleaver length leads in both codes to noticeable improvements in both the bit and the word error rate performance. But the affected SNR regions differ: whereas the CCCs are mostly improved in the "waterfall region", the PSTCs are additionally improved in the "flattening region" (this effect is often referred to as "interleaver gain" in turbo code literature).

Fig. 4 shows that CCCs need a certain number of outer codes ($\nu = 7, 9$) to achieve good performances. Compared with the PSTCs, CCCs with 7 outer codes offer better BER performances at small and moderate SNR. However, for this range of SNR, the classical systematic turbo code ($\rho_u = 1$) outperforms slightly CCCs and PSTCs.

Moreover, it can be observed that the flattening of the BER performance caused by the free-distance asymptote as for the classical turbo code [8], does not appear to be very strong in the CCCs. In addition, increasing the number ν of outer codes from 5 to 7 shifts down the "flattening region" by a factor of 10^{-2}; similar improvements can be expected if ν is further increased. This makes the CCCs with a high number of outer codes preferable for low BER at high SNR. On the other hand, when compared with the classical turbo code, the "flattening region" of the PSTC is considerably lower. Thus, we conclude that the distance properties of PSTCs as well as the distance properties of CCCs are better than those of classical turbo codes.

Comparing the word error rates depicted in Fig. 5 and Fig. 7 shows that both the CCCs and the PSTCs with $\rho_u < 1$ outperform the classical systematic turbo code ($\rho_u = 1$) in the "flattening region", while the performances in the "waterfall region" are only slightly worse. Especially in the "flattening region", the PSTCs provide lower WER than the CCCs.

[3]The BER of long turbo codes employing uniform interleaving can be approximated for high SNR [9].

V. Conclusions

Two new code classes are proposed and compared for medium code lengths. It was shown that the iterative decoding algorithm works not only for systematic, but also for partially systematic (PSTC) or even nonsystematic codes (CCC). PSTCs become stronger by puncturing both the parity and the systematic bits and provide generally the best WER performance. On the other hand, CCCs, especially those with a high number of outer codes, offer competitive BER. In comparison to the classical turbo codes, both codes show better distance properties, which leads to lower error rates in the "flattening region".

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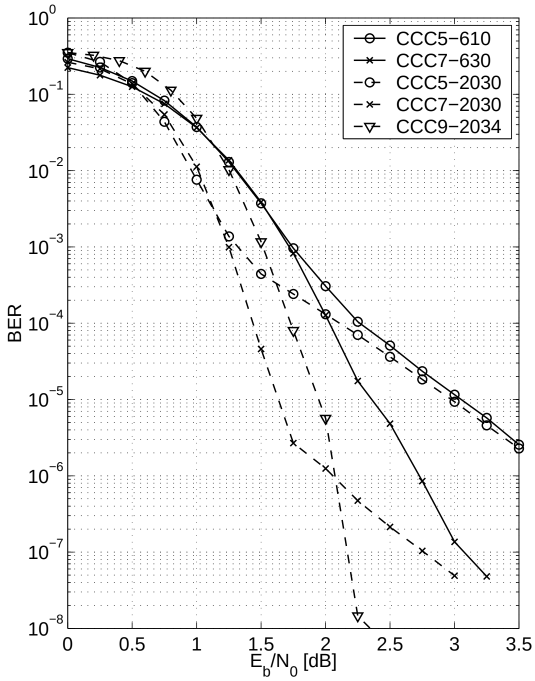

Fig. 4: Bit error rates of convolutional coupled codes CCC_5, CCC_7 and CCC_9; code length $N \approx$ 600 (solid), 2000 (dashed) and rate $R_c \approx 1/2$.

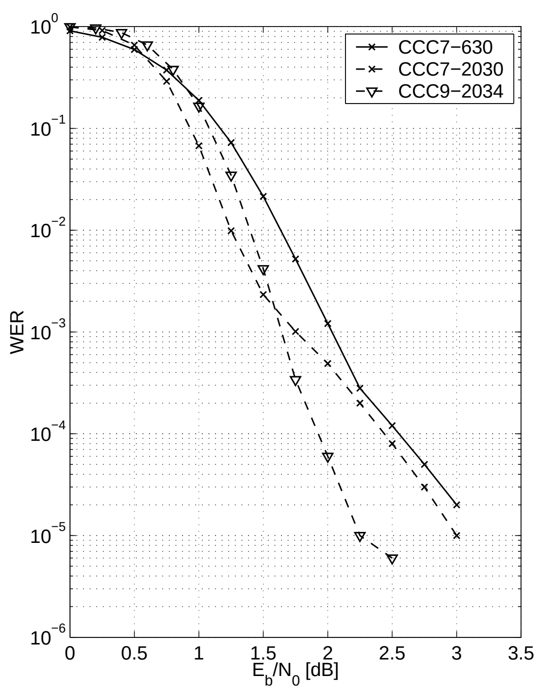

Fig. 5: Word error rates of convolutional coupled codes CCC_7 and CCC_9; code length $N \approx$ 600 (solid), 2000 (dashed) and rate $R_c \approx 1/2$.

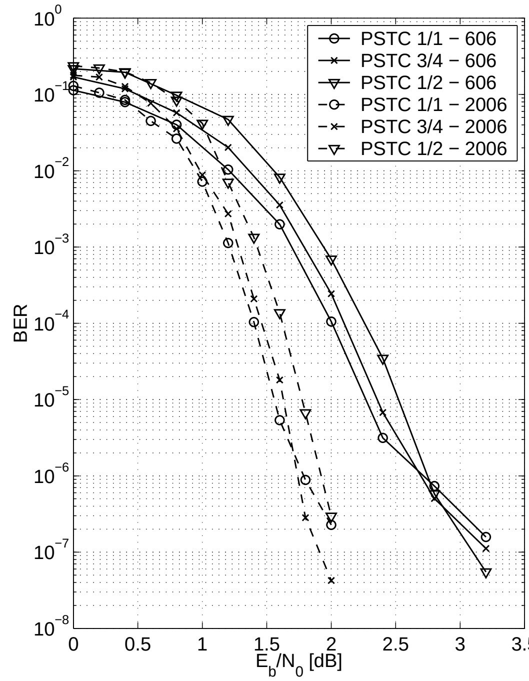

Fig. 6: Bit error rates of partially systematic turbo codes with $\rho_u = 1, 3/4, 1/2$; code lengths $N \approx$ 600 (solid), 2000 (dashed) and rate $R_c \approx 1/2$.

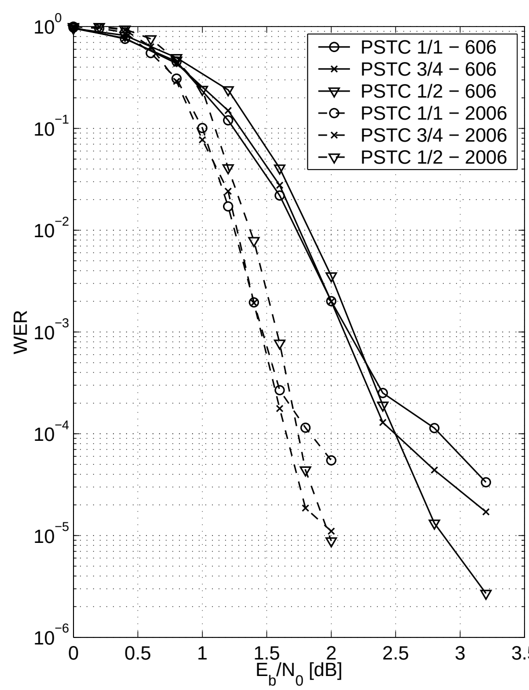

Fig. 7: Word error rates of partially systematic turbo codes with $\rho_u = 1, 3/4, 1/2$; code lengths $N \approx$ 600 (solid), 2000 (dashed) and rate $R_c \approx 1/2$.